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# Engineering Notes

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## Stationary Configurations of a Tetrahedral Tethered Satellite Formation

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### Introduction

ORBITAL dynamics analysis of a connected multibody system is relevant for several missions, including tethered systems, space robots and manipulators, telescopes, space stations, etc. It is important to determine stationary motions of such structures due to their possible use as nominal motions in energy-saving mode. The study of the behavior of multibody systems in the orbital environment continues to grow since its beginning in the 1960s. Sarychev [1] and Wittenburg [2] investigate the equilibria of two connected rigid bodies in circular orbit with respect to the orbital reference frame. Cheng and Liu [3] consider the in-plane dynamics of a three-body system with two equal extreme bodies, examining the equilibrium orientations, bifurcations, and stability with respect to in-plane perturbations. Lavagna and Ercoli Finzi describe planar equilibrium configurations [4] and study their stability [5] for an extended rigid body with two pendulums attached either in a sequence or in a parallel scheme.

Quite frequently, the orbital dynamics of multibody systems is studied for models that approximate connected satellites by an open chain of material points linked by weightless straight rods with spherical hinges. Misra and Modi [6] develop the general three-dimensional formulation for an $n$-link chain. For two- and three-link systems, they determine the libration frequencies and study the control laws. In [7,8] this model is used to determine in-plane equilibrium configurations of a two-link chain and to study their stability. Sarychev [9] finds all spatial equilibrium orientations of a two-link chain in a circular orbit and describes their number as a function of problem parameters. Continued interest in such studies was confirmed recently by Corrêa and Gómez [10], who numerically study the equilibrium configurations of a three-link chain in the plane of the orbit and analyze their stability with respect to in-plane perturbations.

An analytical study of chains with arbitrary number of links can be found in [11,12]. The center of mass of the chain (CMC) moves along a circular orbit. All in-plane equilibrium configurations are described in [11]. All spatial equilibria are listed in [12], in which it is shown that each connecting rod can be oriented with respect to the tangent, normal and binormal to the CMC orbit in one of the following ways:

1) A rod is aligned with the tangent axis.
2) A rod belongs to an NBN group, that is, to a subchain of rods that lies in the plane parallel to normal and binormal so that its center of mass is on the tangent axis.
3) A rod links two NBN groups or an end of an NBN group with the tangent axis.

Some of these equilibrium configurations are actually two-dimensional, with all the system lying in one of the coordinate planes of the orbital reference frame. Yet, there exist essentially three-dimensional configurations, for example, with masses placed at the vertices of a tetrahedron.

Tetrahedral satellite formations are of significant interest due to numerous applications and many projects under development, including NASA research programs in formation flying [13–15]. Tetrahedral configuration has been successfully implemented in the Cluster mission to study three-dimensional structure of the Earth's magnetosphere [16]. Being the simplest spatial configuration, a tetrahedral formation is a natural tool in experiments of this kind, because it enables one to execute simultaneous measurements at points of a large-span three-dimensional basis.

In the present article, we apply the general results of [12] to study tetrahedral equilibrium configurations of a tethered satellite system. We identify the spectrum of spatial equilibrium configurations that can be achieved by varying the masses of the bodies and lengths of the rods. We determine the links that can be replaced with tethers. We study the possibility of controlling such a system and of stabilizing its orientation.

### Statement of the Problem

We study spatial equilibrium configurations of four tethered satellites. We model their dynamics using the following assumptions:

1) The satellites are represented by four material points $A_j$ ($j = 1, \dots, 4$) with respective masses $m_j$ ($j = 1, \dots, 4$).
2) These points are connected by three weightless rigid rods.
3) The junctions are spherical hinges.
4) The Earth's gravitational field is central Newtonian.
5) The CMC moves along a circular orbit of radius $r_*$ with angular velocity $\omega$.
6) The lengths of the rods are small compared with $r_*$.

We describe the dynamics of this system with respect to the right-hand orbital reference frame $Oxyz$. Its origin $O$ is the CMC, the $z$ axis

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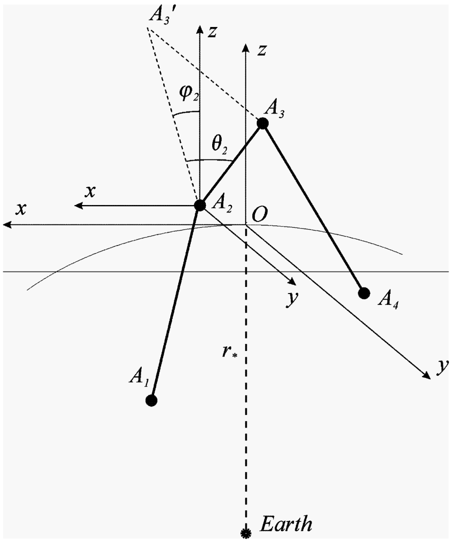



**Fig. 1 Reference frames and orientation angles.**

is along the local vertical, the $x$ axis is tangent to the CMC orbit and points in the direction of the CMC velocity, and the $y$ axis is normal to the plane containing the CMC orbit. We also use other reference frames $A_k xyz$ with axes parallel to those of the $Oxyz$ frame and the origins at points of the chain.

The orientation of the rod $a_k = A_k A_{k+1}$ $(k = 1, \ldots, 3)$ is described by two angles: $\theta_k$ is the angle between the rod and the $x$–$z$ plane $(-\pi/2 \leq \theta_k \leq \pi/2)$ and $\phi_k$ is the angle between the projection of this rod onto the $x$–$z$ plane and the $z$ axis $(-\pi < \phi_k \leq \pi)$ (Fig. 1). The components of the vector $\boldsymbol{a}_k = \overrightarrow{A_k A_{k+1}}$ are

$$a_{kx} = \xi_k = a_k \cos\theta_k \sin\phi_k, \qquad a_{ky} = \eta_k = a_k \sin\theta_k$$
$$a_{kz} = \zeta_k = a_k \cos\theta_k \cos\phi_k, \qquad k = 1, 2, 3$$

We denote by $x_1$, $y_1$, and $z_1$ the coordinates of the point $A_1$ in the orbital frame. The coordinates of all other points $A_k$ are

$$x_{k+1} = x_1 + \sum_{s=1}^{k} \xi_s, \qquad y_{k+1} = y_1 + \sum_{s=1}^{k} \eta_s$$
$$z_{k+1} = z_1 + \sum_{s=1}^{k} \zeta_s, \qquad k = 1, 2, 3$$

The coordinates of the CMC in this frame are

$$x_* = y_* = z_* = 0$$

so we can use the relations

$$\sum_{k=1}^{4} m_k x_k = M x_* = 0, \qquad \sum_{k=1}^{4} m_k y_k = M y_* = 0$$
$$\sum_{k=1}^{4} m_k z_k = M z_* = 0$$

to calculate $x_1$, $y_1$, and $z_1$ and then express the coordinates of the points $A_k$ in terms of six generalized coordinates $\phi_k$ and $\theta_k$ $(k = 1, 2, 3)$, where

$$M = \sum_{1}^{4} m_k$$

is the total mass of the system.

The kinetic energy $T$ of the system is

$$T = \sum_{k=1}^{4} \frac{m_k}{2} \left\{ [\dot{x}_k + \omega(z_k + r_*)]^2 + \dot{y}_k^2 + (\dot{z}_k - \omega x_k)^2 \right\}$$

Its potential energy is

$$V = -\omega^2 r_*^3 \sum_{k=1}^{4} \frac{m_k}{r_k} \tag{1}$$

where

$$r_k = \sqrt{x_k^2 + y_k^2 + (r_* + z_k)^2}$$

is the distance between $A_k$ and the center of the Earth. Approximating the potential energy up to terms of the second order in the small ratios $a_k/r_*$, one can use Eq. (1) to get

$$V = -\omega^2 r_*^3 \sum_{k=1}^{4} m_k \Big[ x_k^2 + y_k^2 + (r_* + z_k)^2 \Big]^{-1/2}$$
$$= -\omega^2 r_*^2 \sum_{k=1}^{4} m_k \left( 1 - \frac{z_k}{r_*} - \frac{x_k^2 + y_k^2 - 2z_k^2}{r_*^2} \right)$$
$$= -M\omega^2 r_*^2 + \frac{\omega^2}{2} \sum_{k=1}^{4} m_k \left( x_k^2 + y_k^2 - 2z_k^2 \right)$$

## Equilibrium Equations

All equilibria of the chain correspond to solutions

$$\phi_k = \phi_{k0} = \text{const}, \qquad \theta_k = \theta_{k0} = \text{const}$$

of the equations

$$\frac{\partial(T_0 - V)}{\partial\phi_k} = 0, \qquad \frac{\partial(T_0 - V)}{\partial\theta_k} = 0, \qquad k = 1, \ldots, n$$

where $T_0$ is the kinetic potential of the system: that is, the terms in the expression of the kinetic energy that do not depend on generalized velocities $\dot{\phi}$ and $\dot{\theta}$. The resulting equations of equilibrium are

$$\xi_k(\mathbf{b}_k \cdot \mathbf{z}) = 0$$
$$3\cos\phi_k \sin\theta_k(\mathbf{b}_k \cdot \mathbf{z}) + \cos\theta_k(\mathbf{b}_k \cdot \mathbf{y}) = 0, \qquad k = 1, 2, 3 \tag{2}$$

where $\mathbf{z} = (\zeta_1, \zeta_2, \zeta_3)^T$, $\mathbf{y} = (\eta_1, \eta_2, \eta_3)^T$, and

$$\mathsf{B} = \begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_3 \end{pmatrix} = \frac{1}{M^2} \begin{pmatrix} m_1(m_2 + m_3 + m_4) & m_1(m_3 + m_4) & m_1 m_4 \\ m_1(m_3 + m_4) & (m_1 + m_2)(m_3 + m_4) & (m_1 + m_2)m_4 \\ m_1 m_4 & (m_1 + m_2)m_4 & (m_1 + m_2 + m_3)m_4 \end{pmatrix}$$

## Tetrahedral Equilibrium Configurations

Equations (2) are a particular case of those studied in [12]. Here, we examine only tetrahedral configurations, which are shown to be of two types.

Tetrahedral equilibria of type 1 correspond to configurations with one end rod of the chain aligned with $Oy$ and the other aligned with the $Oz$ axis, so that their centers of mass lie on the $Ox$ axis (Fig. 2a). The geometry in this case is pretty obvious, and the proper choice of the satellites' mass ratio and length of the rods can provide a wide variety of equilibrium configurations. For example, the tetrahedron formed is ideal when all satellites have equal masses and all rods have equal lengths. However, all configurations of type 1 include two satellites at the same vertical, which can be a disadvantage for some missions because of shadowing.

A simple analysis of forces in the rods leads to the following conclusions: the rod aligned with the local vertical is stretched, the rod directed along the normal to the orbit is compressed, and the one in the middle is not loaded. Consequently, in the system shown in Fig. 2a, the link $A_1A_2$ should be rigid, whereas the rod $A_3A_4$ can be replaced with a flexible tether.

Tetrahedral configurations of type 2 have a different geometry. A chain of three satellites connected by two rods belong to a plane parallel to $Oyz$. Without loss of generality, we assume that these satellites are $A_1, A_2$, and $A_3$. The third rod connects this structure with the last satellite $A_4$ situated on the $Ox$ axis (Fig. 2b). Satellite $A_4$ can be placed at any point of the $Ox$ axis: its position depends only on the length of the connecting rod $A_3A_4$. The other three satellites $A_1$, $A_2$, and $A_3$ form an NBN group situated in the plane orthogonal to the CMC velocity. Their positions are described by the angles $\theta_1$ and $\theta_2$ that satisfy the equations

$$\begin{aligned} &3\sin\theta_1[(m_2+m_3)a_1\cos\theta_1 + m_3a_2\cos\theta_2] \\ &\quad + \cos\theta_1[(m_2+m_3)a_1\sin\theta_1 + m_3a_2\sin\theta_2] = 0 \\ &3\sin\theta_2[m_1a_1\cos\theta_1 + (m_1+m_2)a_2\cos\theta_2] \\ &\quad + \cos\theta_2[m_1a_1\sin\theta_1 + (m_1+m_2)a_2\sin\theta_2] = 0 \end{aligned} \tag{3}$$

where $-\pi \le \theta_1, \theta_2 \le \pi$. We look for three-dimensional solutions of type 2, and so necessarily $\sin\theta_1 \ne 0$ (otherwise, we would also have $\sin\theta_2 = 0$, and the chain would lie in the orbit plane). Likewise, $\sin\theta_2 \ne 0$, $\cos\theta_1 \ne 0$, and $\cos\theta_2 \ne 0$. We divide the first equation of Eq. (3) by $m_3a_2\sin\theta_1\cos\theta_1$ and the second one by $m_3a_2\sin\theta_2\cos\theta_2$. Using new variables and parameters

$$p = \frac{\cos\theta_2}{\cos\theta_1}, \qquad r = \frac{\sin\theta_2}{\sin\theta_1}, \qquad \lambda = \frac{a_1}{a_2}$$
$$\mu_1 = \frac{m_1}{m_3}, \qquad \mu_2 = \frac{m_2}{m_3} \tag{4}$$

we can transform system (3) to

$$\begin{aligned} &3p + r + 4\lambda(\mu_2+1) = 0 \\ &\mu_1\lambda\left(\frac{3}{p} + \frac{1}{r}\right) + 4(\mu_1+\mu_2) = 0 \end{aligned} \tag{5}$$

System (5) has solutions

$$p_{1,2} = -\frac{\lambda}{3(\mu_1+\mu_2)}\left(3\mu_1 + 2\mu_2 + 2\mu_1\mu_2 + 2\mu_2^2 \pm \sqrt{D}\right)$$
$$r_{1,2} = -\frac{\lambda}{\mu_1+\mu_2}\left(\mu_1 + 2\mu_2 + 2\mu_1\mu_2 + 2\mu_2^2 \mp \sqrt{D}\right)$$

where

$$D = \mu_2(1+\mu_1+\mu_2)\left(3\mu_1 + 4\mu_2 + 4\mu_1\mu_2 + 4\mu_2^2\right) > 0$$

because $\mu_1$ and $\mu_2$ are positive. Therefore, system (5) has two pairs of solutions. For each pair $(p, r)$, one can use Eq. (4) to obtain

$$\cos\theta_2 = p\cos\theta_1, \qquad \sin\theta_2 = r\sin\theta_1$$
$$(\cos\theta_2)^2 + (\sin\theta_2)^2 = p^2\cos^2\theta_1 + r^2\sin^2\theta_1 = 1$$

and so

$$\sin^2\theta_1 = \frac{1-p^2}{r^2-p^2}$$

If

$$0 < \frac{1-p^2}{r^2-p^2} < 1$$

there exist two values of $\sin\theta_1$. Each one corresponds to two angles $\theta_1$ $(-\pi \le \theta_j \le \pi)$. Hence, for given parameters of the system, there exist, at most, eight solutions of type 2. Similar results were obtained in [9].

An important issue to be addressed here is the geometry of equilibria that can be achieved by an appropriate choice of system parameters. The restrictions on angles $\theta_1$ and $\theta_2$ are determined by system (3). One can solve it with respect to the parameters $\mu_1$ and $\mu_2$ and find

$$\mu_1 = \frac{\cos\theta_2\sin\theta_2(3\cos\theta_2\sin\theta_1 + \cos\theta_1\sin\theta_2 + 4\lambda\cos\theta_1\sin\theta_1)}{\lambda\cos\theta_1\sin\theta_1(4\cos\theta_2\sin\theta_2 + \lambda\cos\theta_2\sin\theta_1 + 3\lambda\cos\theta_1\sin\theta_2)}$$
$$\mu_2 = -\frac{3\cos\theta_2\sin\theta_1 + \cos\theta_1\sin\theta_2 + 4\lambda\cos\theta_1\sin\theta_1}{4\lambda\cos\theta_1\sin\theta_1}$$

and

$$\begin{aligned} \mu_1 &= -\mu_2\frac{4\cos\theta_2\sin\theta_2}{4\cos\theta_2\sin\theta_2 + \lambda\cos\theta_2\sin\theta_1 + 3\lambda\cos\theta_1\sin\theta_2} \\ &= -\mu_2\frac{4pr}{4pr + \lambda p + 3\lambda r} \end{aligned}$$

Because $\mu_1 > 0$ and $\mu_2 > 0$, we arrive at two conditions that define the possible configurations of the chain:

$$\frac{3}{p} + \frac{1}{r} + \frac{4}{\lambda} < 0, \qquad 3p + r + 4\lambda < 0$$

Figures 3–8 show the attainable geometries for tetrahedral configurations of type 2 for the indicated values of $\lambda = a_1/a_2$. Axis $\theta_1$ is horizontal and axis $\theta_2$ is vertical. It suffices to study the interval $0 < \lambda \le 1$. Solutions exist only for $\theta_1\theta_2 < 0$. We show the regions $0 < \theta_1 < \pi$ and $-\pi < \theta_2 < 0$, because for $-\pi < \theta_1 < 0$ and $0 < \theta_2 < \pi$, the picture is the same. The shadowed regions are the domains in the plane $\theta_1$–$\theta_2$, in which equilibrium configurations exist. Note that attainable geometries are confined to small regions on the plane $\theta_1$–$\theta_2$.

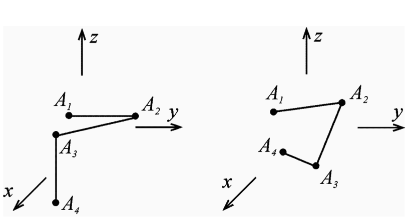
When the ratio between the lengths of the rods is fixed, the area formed by the NBN rods increases as

$$\theta_1 - \theta_2 \to \pm\pi/2, \quad \pm 3\pi/2$$

One can see that this happens only when the angles $(\theta_1, \theta_2)$ converge to one of the following values:

$$(\pm\pi, \mp\pi/2), \qquad (\pm\pi/2, \mp\pi), \qquad (\pm\pi/2, 0), \qquad (0, \pm\pi/2)$$

In these cases, one of the rods approaches the vertical position, whereas the other tends to the horizontal one. All these cases




a) b)

**Fig. 2 Tetrahedral equilibrium configurations.**

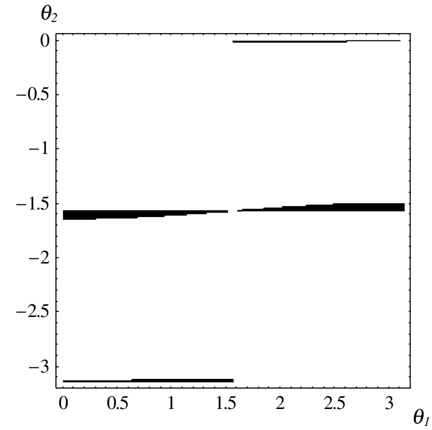


**Fig. 3 λ = 0.1.**

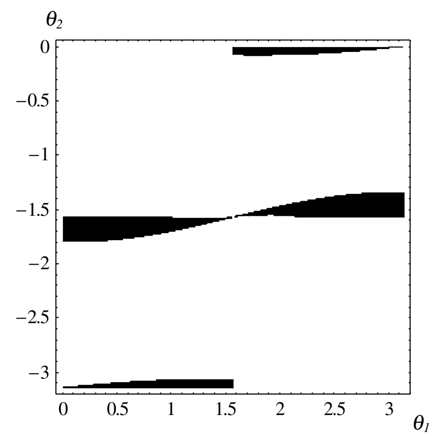


**Fig. 4 λ = 0.3.**

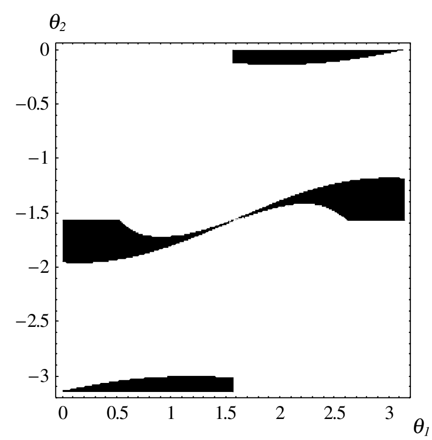


**Fig. 5 λ = 0.5.**

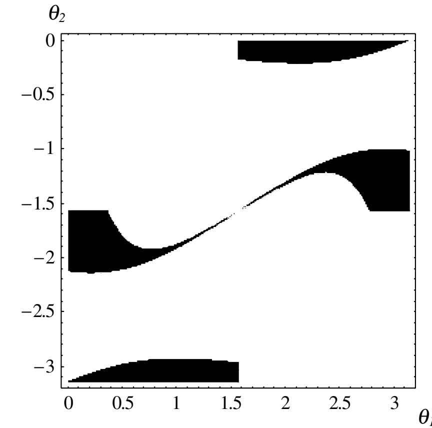


**Fig. 6 λ = 0.7.**

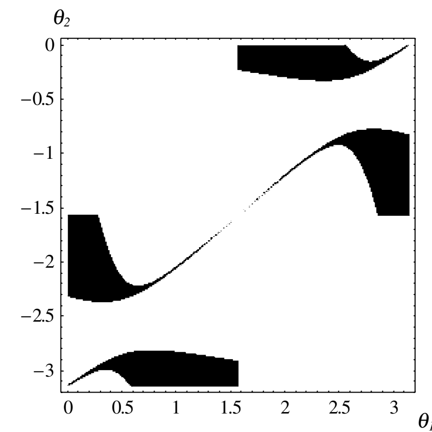


**Fig. 7 λ = 0.9.**

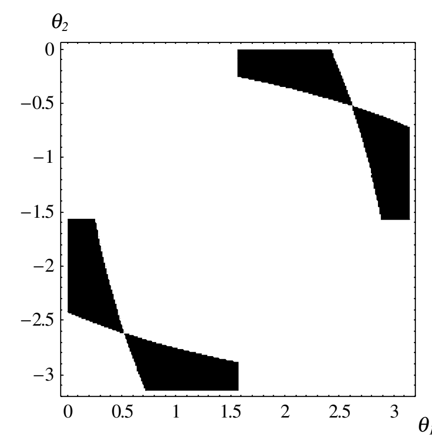


**Fig. 8 λ = 1.**

correspond to $\mu_2 \to \infty$ with $\mu_1 = \mathcal{O}(1)$. These relations mean that in the NBN group, the mass of the central satellite $A_2$ should be much greater than those of $A_1$ and $A_3$.

Analysis of forces in the rods for type-2 configurations shows that a rod is compressed if it crosses the orbit plane, and so it should be rigid. If a rod does not intersect the orbit plane, it is stretched and can be replaced with a flexible tether. Finally, the rod that links NBN group with $A_4$ is unloaded.

## Stability and stabilization

The stability analysis performed for tetrahedral equilibria of an open chain in a circular orbit shows that the configurations of both types are unstable.

Consider first an equilibrium of type 1 (Fig. 2a). This system can be regarded as two equilibrated subsystems (rod $A_1A_2$ normal to the orbit plane and rod $A_3A_4$ directed along the local vertical) with one geometrical constraint (connecting link $A_2A_3$). Rod $A_1A_2$ is stable [17]. Rod $A_3A_4$ is unstable. Because the angular dynamics of the rod perpendicular to the orbit plane are locally equivalent to the dynamics of an unstable spherical pendulum, the instability degree of this system (i.e., the number of imaginary frequencies in the normal form of the system) is equal to two. However, it is well known [18] that it is impossible to stabilize a mechanical system with an instability degree greater than one by imposing one geometrical constraint. Thus, all equilibria of type 1 are unstable.

Consider now an equilibrium of type 2 (Fig. 2b). The chain can be divided into two equilibrated subsystems: triangle $A_1A_2A_3$ in the plane orthogonal to the orbit, with the instability degree equal to at least one, and point $A_4$ moving along the circular orbit, also with an instability degree equal to one. Adding a new link, we get a system with the instability degree of at least one. Thus, configurations of both types 1 and 2 are unstable.

One can stabilize both configurations by control applied to a single generalized coordinate. Consider, for example, the case of identical satellites and rods. If it is possible to control a generalized force corresponding to some generalized coordinate (say, an angle $\theta_k$), we get a control system that, after linearization, satisfies the Kalman controllability condition. Therefore, the configuration can be stabilized in the neighborhood of the equilibrium position by a feedback that is linear with respect to generalized coordinates and velocities.

## Conclusions

We study tetrahedral equilibrium configurations of a chain that consists of four satellites connected by three rigid weightless rods. These configurations are shown to be of two different types. In an equilibrium of the first type, one end rod occupies the vertical position and the other is normal to the orbit plane. The limitation of these configurations is that two satellites are necessarily located at the same vertical. The second type of equilibrium corresponds to two rods in the plane orthogonal to the CMC velocity vector. These equilibria provide a greater variety of satellites' positions with respect to the Earth. Although both types of equilibrium configurations are shown to be unstable, they can be stabilized by a control torque applied to one of the rods.

The attainable tetrahedral geometries are described in terms of the system parameters. We study the possibility to increase the formation volume.

Analysis of the forces in the links indicates that the rods that are either aligned with the local vertical or do not intersect the orbit plane are stretched and can be replaced with flexible tethers. The rods that cross the orbit plane are compressed and should be rigid. All the rods with a nonzero component along the tangent to the CMC orbit are not loaded. So all tetrahedral equilibrium configurations contain one unloaded link and at least one compressed rod. Therefore, they cannot be implemented using only tethers: at least one link should be rigid.

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